Multiple Base Cases

- Start with one Base case
- Check if you fall off the bottom of ladder
  - Add base case

1st induction
Base case

Falling off the ladder
F(n)
1. If n ≤ 1: return n
2. return 5F(n-1) - 6F(n-2)

Q: Prove this algorithm returns $3^n - 2^n$ for all $n \geq 0$.

Let $P(n)$ be the predicate $F(n)$ returns $3^n - 2^n$. We will prove $P(n)$ is true for all $n \geq 0$, using strong induction.

**Base cases**: We will show $P(0)$ and $P(1)$. When the input is 0, we return 0. Since $3^0 - 2^0 = 1 - 1 = 0$, this is correct. When the input is 1, we return 1. Since $3^1 - 2^1 = 3 - 2 = 1$, this is correct.

**Inductive step**: Let $k \geq 1$. Assume $P(j)$ is true for all $j$ such that $0 \leq j \leq k$. We will prove $P(k+1)$.

We need to prove $P(0)$ and $P(1)$. Otherwise when trying to prove $P(2)$, look at $f(2-1) = f(1)$ and $f(2-2) = f(0)$, need to assume these output correctly.