Calculating Probability

What is the probability of (2 dice sum to 9)?

1. Define the sample space
   - think big!
   - set of all possible outcomes

   \[ S = \{ (1,1), (1,2), (1,3), \ldots, (6,6) \} \]
   \[ = \{ 1, 2, 3, 4, 5, 6 \}^2 \]

2. Define the event
   - subset of sample space you care about

   \[ E = \{ (6,3), (3,6), (5,4), (4,5) \} \]
3. Ask: are all elements of sample space equally likely?

Yes

\[ \Pr(E) = \frac{|E|}{|S|} \]

every outcome is equally likely

|E| = 4

|S| = 6 \times 6

(product rule)

No

\[ \Pr(E) = \sum_{i \in E} \Pr(i) \]

Sum over all elements in E

\[ \sum_{i \in S} \Pr(i) = 1 \]

Useful fact: \( \Pr(S-E) = 1 - \Pr(E) \)

not E (sometimes easier to calculate not E)
Q: Lottery where a 4-digit number is chosen randomly. What is the size of the sample space?

A) 40  B) 10,000  C) (10)^4

Use product rule:
10·10·10·10

Suppose you win some money if you get 3 of 4 numbers matching.

What is the probability you don't win if you buy 1 ticket with # 1313.

1. Sample Space: \( \{1,2,3,...,10\}^4 \)
2. Event: (I'll calculate probability of winning and then do \( 1 - \Pr(E) \).)

\[ E = \{ x: x \in \{1,2,3,...,10\}^4 \land x \text{ agrees with 1313 in at least 3 positions} \} \]
3. Use formula \( |E|/|S| \) b/c all outcomes are equally likely.
\[ |E| \Rightarrow \text{agree in all 4 positions} \]

\[ 1 + 4 \cdot 9 = 37 \]

\[ \Pr(E) = 1 - \frac{37}{1000} \]

\[ = 0.9963 \]