Goal: Use counting rules to solve problems

Counting

Q: Why is counting important in Computer Science?

A:  
   • Security: count \# of possible passwords
   • Alg: count steps of algorithm
     - count space (memory) used by algorithm
     - count time of algorithm
   • Network: count connections in a network
   • Architecture: count ways of distributing tasks to processors

Product Rule: If a procedure can be broken down into two tasks, with \( n_1 \) ways to do the first task and \( n_2 \) ways to do the second, then there are \( n_1 \times n_2 \) ways to do the procedure. (If \( k \) tasks, multiply ways to do first by ways to do second by ways to do 3rd \ldots up to \( k \).

Subtraction Rule

If you can do a task \( n_1 \) ways or \( n_2 \) ways, then the total number of ways to do a task is \( n_1 + n_2 \) minus the number of ways common to the two approaches.
Q: Suppose you and your best friend are picking into the Fermi house. There are 20 singles left and 3 doubles. If you both choose singles or both share a double, how many options of room choices do you have?

1st Choice:

Single or Double

\[
\begin{align*}
\text{options with both single and double} & \\
\text{# single options} & + \ # \text{ double options} - 0
\end{align*}
\]

I choose 20 and Friend chooses 19.

\[20 \times 19 = 383\]

(because I've used up one option)
See slides for problems without solutions

Q: How many 5-bit strings start with 1 or end with 00?

Start with 1 or End with 00 or Both?

\[
\begin{align*}
\overline{1} \overline{1} \overline{1} \overline{1} \overline{1} \\
\overline{0} \overline{0} \\
\text{Choose bit and choose bit and choose bit and choose bit} \\
2 \times 2 \times 2 \times 2 \times 2
\end{align*}
\]

\[
\begin{align*}
\overline{0} \overline{0} \overline{0} \\
\overline{1} \overline{1} \overline{1} \\
\text{Choose bit and choose bit} \\
2 \times 2 \times 2
\end{align*}
\]

\[
\begin{align*}
6 + 8 - 4 = 20
\end{align*}
\]
Q: Suppose you are the track coach and you want to test different options for the 4 person relay team. You have 5 runners: A, B, C, D, and E. If A is on the team, you want her in one of the first 2 positions. If B is on the team, you want her in one of the last 2 positions. How many options will you need to test to find the optimal order?

Pick A's Position

A --

B last, 2
(no A)

or

B in 3
(no both)

and

Pick Remaining Positions

A --

B, A

1st remaining

Pick A and B's position

2nd remaining

3.2.1

or

B in 3

or

B, in 4

3.2.1

3 options
(C, D, E)

Pick A and Pick B

2 x 2

2.6 + (6+6) + 4x6 = 48 options