Name ____________________________

CS200 - Midterm Review Questions

1. This is a little easier than what would be on exam, but good practice for setting up the techniques. Prove that if n is even, $n + 4$ is even, using direct proof, prove by contrapositive, proof by contradiction.

2. These are a little harder than what would be on exam, but good practice for setting up the techniques:

   (a) Prove using a proof by cases that $\forall n \in \mathbb{Z}$, $n^2 \geq n$.

   (b) Now prove using proof by contradiction and using cases that $\forall n \in \mathbb{Z}$, $n^2 \geq n$. Hint: think about dividing both sides by n and $-n$.

   Solution

   (a) In the case $n \geq 1$, then multiplying both sides of the inequality by n, we have $n^2 \geq 1$. In the case $n = 0$, we have $n^2 = n = 0$. In the case that $n \leq -1$, we have n^2 is positive but n is negative, so $n^2 \geq n$.

   (b) Suppose there exists some $n \in \mathbb{Z}$ such that $n^2 < n$. n can't be zero, because it is not true that $0 < 0$. But if $n \geq 1$, then we can divide both sides by n to get $n < 1$, which is a contradcition. If $n \leq 1$, then we can divide both sides by n, and the inequality switched direction, and we have $n > 1$, a contradiction. Thus, in all cases we have a contradiction, and so the statement must be true.

3. Let $G(x, y)$ be the predicate, x is the grandmother of y.

   (a) Let Y be the statement: "No one has more than two grandmothers." Then $Y \equiv$

   (b) Let Q be the statement "Everyone has exactly two grandmothers." The $Q \equiv$

4. You meet a group of 50 orcs. You know orcs are either honest or corrupt. Suppose you know that at least one of the orcs is honest. You also know that given any two of the orcs, at least one is corrupt. Can you figure out how many of the orcs are corrupt and how many are honest? If G is the set of orcs, and $C(g)$ is the predicate, "orc g is corrupt," can you express these statements ("given any two orcs, at least one is corrupt," "given any two orcs, at least one is corrupt") using math?

5. Who is in the common room? Either Kevin or Sebin, or both, are there. Either Alexei or Vijay, but not both, are there. If Erin is there, so is Alexei. Vijay and Kevin are both there, or neither is there. If Sebin is there, so are Erin and Kevin.

Solution We have $K \vee S$, $A = \neg V$, $E \to A$, $V \leftrightarrow K$, $S \to E \wedge K$, where the letter means that person is in the room. You could do a truth table, but let's try to work it out without. Let's consider first the case that S is true. Then E and V are both true by $S \to E \wedge K$,. Then A is true by $E \to A$. But this contradicts that $A = \neg V$. Thus S is false. Then by $K \vee S$, we have K is true. But now $V \leftrightarrow K$, so V is true. Thus by $A = \neg V$ we have A is false. Finally, since A is false, E can't be true, or otherwise $E \to A$ is false. Thus, we find Kevin and Vijay are in the room. Seem OK?