

CS200 - Worksheet 5

1. Let $T(n)$ be the number of strings in $\{0, 1, 2\}^n$ that do *not* contain two consecutive zeros. Write a recurrence relation for $T(n)$
2. **[Challenge]** Let $T(n)$ be the number of strings in $\{0, 1, 2\}^n$ that do not have 2 consecutive 0's or 2 consecutive 1's. Create a recurrence relation for $T(n)$.
3. Suppose you have a coin that has a changing probability of getting heads. When you toss it the i th time, the probability of getting heads is $1/2^i$. If you flip the coin an infinite number of times, how many heads would you expect to see?
4. **[Challenge]** Let $[n] = \{1, 2, 3, \dots, n\}$. Given a permutation of the elements of $[n]$, an inversion is an ordered pair (i, j) with $i, j \in [n]$, such that $i < j$, but j precedes i in the permutation. For instance consider the set $[5]$, and the permutation $(3, 5, 1, 4, 2)$. There are six inversions in this permutation:

$$(1, 3), (1, 5), (2, 3), (2, 4), (2, 5), (4, 5). \tag{1}$$

If a permutation is uniformly at random from among all permutations, what is the expected number of inversions? (Hint - use indicator random variables! To figure out the probability of the indicator event happening, try a small example, like $[3]$.)