(Taken from *Discrete Mathematics, an Open Introduction* by Levin). Read the following proofs of the statement: If $ab$ is even, then $a$ or $b$ is even. What do you notice? Especially consider similarities and differences regarding language, style, and structure. What words are used repeatedly, and what do those words signal to the reader?

1. Suppose $a$ and $b$ are odd. That is, $a = 2k + 1$ and $b = 2m + 1$ for some integers $k$ and $m$. Then

$$ab = (2k + 1)(2m + 1) = 4km + 2k + 2m + 1 = 2(2km + k + m) + 1.$$  \hfill (1)

Therefore, $ab$ is odd.

2. Assume that $a$ or $b$ is even. Suppose it is $a$, since the case where $b$ is even will be identical. That is, $a = 2k$ for some integer $k$. Then

$$ab = (2k)b = 2(kb).$$  \hfill (2)

Therefore $ab$ is even.

3. Suppose that $ab$ is even but $a$ and $b$ are both odd. Namely, $a = 2k + 1$ and $b = 2j + 1$ for some integers $k$, and $j$. Then

$$ab = (2k + 1)(2j + 1) = 4kj + 2k + 2j + 1 = 2(2kj + k + j) + 1.$$  \hfill (3)

But this means that $ab$ is odd, which contradicts our premise. Thus $a$ and $b$ can not both be odd.

4. Assume $ab$ is even. Namely, $ab = 2n$ for some integer $n$. Then there are two cases: $a$ must be either even or odd. If it is even then the statement is true. If it is odd, then $a = 2k + 1$ for some integer $k$. Then

$$2n = (2k + 1)b = 2kb + b$$

$$2(n - kb) = b.$$  \hfill (4)

Therefore, $b$ must be even, and the statement is true.