CS200 - Worksheet 2

(Taken from *Discrete Mathematics, an Open Introduction* by Levin). Read the following proofs of the statement: If ab is even, then a or b is even. What do you notice? Especially consider similarities and differences regarding language, style, and structure. What words are used repeatedly, and what do those words signal to the reader?

1. Suppose a and b are odd. That is, $a = 2k + 1$ and $b = 2m + 1$ for some integers k and m. Then

$$\begin{aligned} ab &= (2k+1)(2m+1) \\ &= 4km + 2k + 2m + 1 \\ &= 2(2km + k + m) + 1. \end{aligned} \tag{1}$$

   Therefore, ab is odd.

2. Assume that a or b is even. Suppose it is a, since the case where b is even will be identical. That is, $a = 2k$ for some integer k. Then

$$ab = (2k)b = 2(kb). \tag{2}$$

   Therefore ab is even.

3. Suppose that ab is even but a and b are both odd. Namely, $a = 2k + 1$ and $b = 2j + 1$ for some integers k, and j. Then

$$\begin{aligned} ab &= (2k+1)(2j+1) \\ &= 4kj + 2k + 2j + 1 \\ &= 2(2kj + k + j) + 1. \end{aligned} \tag{3}$$

   But this means that ab is odd, which contradicts our premise. Thus a and b can not both be odd.

4. Assume ab is even. Namely, $ab = 2n$ for some integer n. Then there are two cases: a must be either even or odd. If it is even then the statement is true. If it is odd, then $a = 2k + 1$ for some integer k. Then

$$\begin{aligned} 2n &= (2k+1)b \\ &= 2kb + b \\ 2(n - kb) &= b. \end{aligned} \tag{4}$$

   Therefore, b must be even, and the statement is true.