1. Describe the following sets in roster notation (list the first few elements). If the set is also “famous” give its symbol.

(a) $A = \{2^x : x \in \mathbb{N}\}$
(b) \( B = \{ x : x \text{ is even and } x \in \{1, 3, 5\} \} \)
(c) \( C = \{ x \geq 0 : x \text{ is even or } x \text{ is odd} \} \)

Solution
(a) \( A = \{1, 2, 4, 8, 16, \ldots \} \)
(b) \( B = \emptyset \)
(c) \( C = \{0, 1, 2, 3, 4, \ldots \} = \mathbb{N} \)

2. Let \( A = \{1, 2\} \) and \( B = \{1, 2, 3\} \)

(a) What is \( |A \times B|\)?
(b) Is \( A \subset B\)?
(c) Is \( A \subseteq B\)?
(d) Is \( A \subset A\)?
(e) What is \( A \setminus B\)?
(f) What is \( A \cup B\)?
(g) What is \( A \cap B\)?

Solution
(a) \( A \times B = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3)\} \), so \( |A \times B| = |A| \times |B| = 6 \).
(b) Yes. Both 1 and 2 are elements of \( B \).
(c) Yes. Both 1 and 2 are elements of \( B \).
(d) No. \( \subset \) cannot be used when the two sets are not equal.
(e) \( \emptyset \).
(f) \( B \). \( B \) already contains all the elements of \( A \), so adding those elements doesn’t do anything.
(g) \( A \). The elements of \( A \) are in both. Only 3 \( \in B \) but 2 \( \notin A \).

3. Let \( A \) and \( B \) be sets with \( |A| = |B| \) such that \( |A \cup B| = 7 \) and \( |A \cap B| = 3 \). What is \( |A| \)? Explain.

Solution \( 7 = |A \cup B| = |A \cap B| + |A \setminus B| + |B \setminus A| \). But \( |A \setminus B| = |B \setminus A| \) because \( |A| = |B| \), so \( |A \setminus B| = 2 \) and \( |A| = |A \cap B| + |A \setminus B| = 5 \).

4. Find sets \( A \) and \( B \) such that \( A \subset B \) and \( A \in B \).

Solution For example, \( A = \{1, 2\} \), \( B = \{1, 2, 3, 4, \{1\}, 5\} \).