CS200 - Worksheet 1

Use the following definitions to answer the questions below (image taken from *Discrete Mathematics, an Open Introduction* by Levin):

Set Theory Notation

$\{,\}$	We use these **braces** to enclose the elements of a set. So $\{1,2,3\}$ is the set containing 1, 2, and 3.
$:$	$\{x : x > 2\}$ is the set of all x **such that** x is greater than 2.
$\in$	$2 \in \{1,2,3\}$ asserts that 2 is **an element of** the set $\{1,2,3\}$.
$\notin$	$4 \notin \{1,2,3\}$ because 4 **is not an element of** the set $\{1,2,3\}$.
$\subseteq$	$A \subseteq B$ asserts that A **is a subset of** B: every element of A is also an element of B.
$\subset$	$A \subset B$ asserts that A **is a proper subset of** B: every element of A is also an element of B, but $A \neq B$.
$\cap$	$A \cap B$ is the **intersection of** A **and** B: the set containing all elements which are elements of both A and B.
$\cup$	$A \cup B$ is the **union of** A **and** B: is the set containing all elements which are elements of A or B or both.
$\times$	$A \times B$ is the **Cartesian product of** A **and** B: the set of all ordered pairs (a,b) with $a \in A$ and $b \in B$.
$\setminus$	$A \setminus B$ is A **set-minus** B: the set containing all elements of A which are not elements of B.
$\overline{A}$	The **complement of** A is the set of everything which is not an element of A.
$\|A\|$	The **cardinality (or size) of** A is the number of elements in A.

1. Describe the following sets in roster notation (list the first few elements). If the set is also "famous" give its symbol.

   (a) $A = \{2^x : x \in \mathbb{N}\}$

(b) $B = \{x : x \text{ is even and } x \in \{1, 3, 5\}\}$
(c) $C = \{x \geq 0 : x \text{ is even or } x \text{ is odd}\}$

Solution

(a) $A = \{1, 2, 4, 8, 16, \dots\}$
(b) $B = \{\} = \emptyset$
(c) $C = \{0, 1, 2, 3, 4, \dots\} = \mathbb{N}$

2. Let $A = \{1, 2\}$ and $B = \{1, 2, 3\}$

(a) What is $|A \times B|$?
(b) Is $A \subset B$?
(c) Is $A \subseteq B$?
(d) Is $A \subset A$?
(e) What is $A \setminus B$?
(f) What is $A \cup B$?
(g) What is $A \cap B$?

Solution

(a) $A \times B = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3)\}$, so $|A \times B| = |A| \times |B| = 6$.
(b) Yes. Both 1 and 2 and elements of B.
(c) Yes. Both 1 and 2 and elements of B.
(d) No. $\subset$ can only be used when the two sets are not equal.
(e) $\emptyset$.
(f) B. B already contains all the elements of A, so adding those elements doesn't do anything
(g) A. The elements of A are in both. Only $3 \in B$ but $2 \notin A$.

3. Let A and B be sets with $|A| = |B|$ such that $|A \cup B| = 7$ and $|A \cap B| = 3$. What is $|A|$? Explain.

Solution $7 = |A \cup B| = |A \cap B| + |A \setminus B| + |B \setminus A|$. But $|A \setminus B| = |B \setminus A|$ because $|A| = |B|$, so $|A \setminus B| = 2$ and $|A| = |A \cap B| + |A \setminus B| = 5$.

4. Find sets A and B such that $A \subset B$ and $A \in B$.

Solution For example, $A = \{1, 2\}$, $B = \{1, 2, 3, 4, \{1, 2\}, 5\}$.