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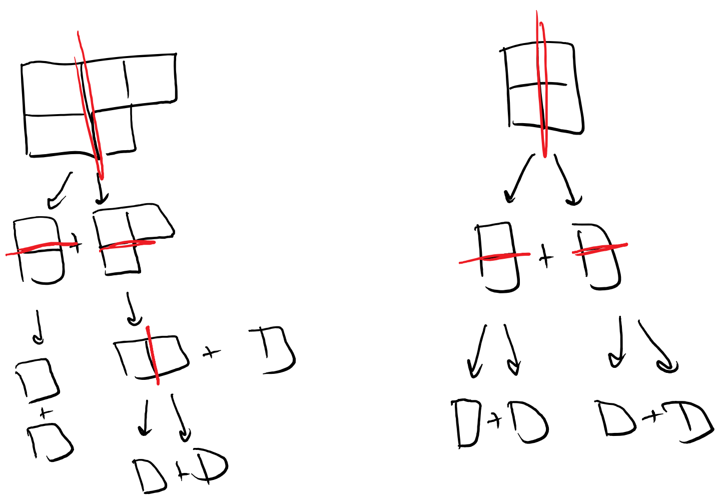
Strong Induction

Q: Suppose you have a bar of chocolate containing n small joined squares. How many times do you have to break the chocolate along a row or column before you have n separate squares?



always seems to be $n-1$!

Proof?

Induction seems good, because after breaking, end up with smaller chocolate bars (smaller problems!)

BUT

smaller bar might not have $n-1$ pieces

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Strong Induction Proof Structure

Set-up

Let $P(n)$ be the predicate ________________________ We will prove $P(n)$ is true for all $n \geq b.c.$

base case

Base-case

Base-case: We prove $P(b.c.)$ is true. . . .

Inductive case

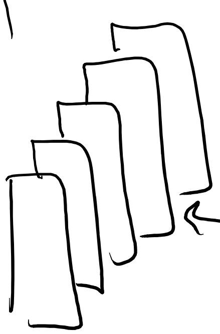
Inductive case: Assume for strong induction that $P(k)$ is true for all $k: 0 \leq k < n$. We now prove $P(n)$ is true. . . .

Rest of proof should not use "k". Use other variables. Should use "n"

Conclusion

By strong induction, we conclude $P(n)$ is true for all $n \geq 0$

Metaphor:



k^{th} domino alone is too light to knock over k+1. Need weight of earlier ones

Base Case: Knock first over

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Q: Prove it takes $n-1$ breaks to reduce an n-square chocolate bar to n individual squares.

A: Let $P(n)$ be the predicate " ." We will prove via strong induction that $P(n)$ is true for $n \in \mathbb{N}$, $n \geq 1$.

Base case: When you have a 1-square chocolate bar, it requires 0 breaks to create 1 individual squares, so $P(1)$ is true.

Inductive Case: We assume for strong induction that $P(k)$ is true for $1 \leq k < n$. We will prove $P(n)$ is true. Since $n > 1$, we can

we can break it into two pieces, one with a squares, and one with b squares, where $a + b = n$, and $1 \leq a < n$, and $1 \leq b < n$. Using our inductive assumption, it requires $(a-1)$ breaks to separate the first piece and $(b-1)$ breaks to separate the second. Adding up all the breaks, we have

$$(a-1) + (b-1) + 1 = a + b - 1 = n-1.$$

total breaks.