Recall Sum/Product/Subtraction

Q: There are 10 singles left in a coffin and you and 2 friends want to pick 3 of them. How many ways could you choose rooms?

A) 30  B) 300  C) 720  D) 1000

Answer: Using product rule \(10 \cdot 9 \cdot 8 = 720\)

New function:
\[ P: \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N} \]
\(P(n, k)\) is the number of \(k\)-permutations of \(n\) elements.

E.g. \(P(10, 3) = 720\).
Q:
What is a permutation?
- An ordering of a set of elements

What is a k-permutation?
- An ordering of a set of k elements

What is a k-permutation of n elements?
- An ordering of a set of k elements where those k are chosen from n elements

What is a formula for P(n, k)? Using product rule:
\[ n \cdot (n-1)(n-2) \ldots (n-k+1) \]
or \[ \prod_{i=n-k+1}^{n} i \]

How many permutations are there of n elements?
- \[ n \cdot (n-1) \cdot (n-2) \cdot (n-3) \ldots \cdot 1 = n! \]
Another way to write $P(n,k)$:

$$10 \cdot 9 \cdot 8 \frac{(7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)}{(7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)} = \frac{10!}{7!}$$

Where $n! = n \cdot (n-1) \cdot (n-2) \cdots \cdot 1 = \prod_{i=1}^{n} i$

So $P(n,k) = \frac{n!}{(n-k)!}$

Q: There are 10 singles left in Coffin and you and 2 friends want to pick 3 of them. Suppose you just want to pick 3 rooms now, and you'll figure out who will stay where later. How many ways could you pick 3 rooms?

A) 30  B) 120  C) 240  D) 360
We know 720 ways if care about order.

So: \((2,3,5), (2,5,3), (3,2,5), (3,5,2), (5,2,3), (5,3,2)\)

My 1 pick 1 pick
Friend 2 pick 2 pick

But if don’t care about order, these are all the same. \(\{2,3,5\}\)

\(
\implies \text{Over counting by a factor of 6 for each set!}
\)

\[
720/6 = 120
\]
Function
\[ C(n, r) = \binom{n}{r} = \text{"n choose r"} \] is the number of sets of \( r \) elements chosen from a set of \( n \) elements.

Fact: \[ \binom{n}{r} = \frac{n!}{r! \cdot (n-r)!} \]

Proof: \( P(n, r) = \binom{n}{r} \cdot P(r, r) \)

\[ \Rightarrow \binom{n}{r} = \frac{P(n, r)}{P(r, r)} = \frac{n!}{(n-r)! \cdot \frac{r!}{1!}} = \frac{n!}{(n-r)! \cdot r!} \]

The number of ways we can order \( r \) things chosen from among \( n \) things is equal to the number of subsets of \( r \) things, times the ways we can order each subset.
Q: If 8 people from a basketball team show up to a game, how many ways are there to form a 5 person team?

A) 40   B) 56   C) 60   D) 112

\[
\frac{8!}{5!3!} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 8 \cdot 7 = 56
\]