CS200 - Problem Set 5
Due: Monday, Oct. 16 to submission server before class
Please read the sections of the syllabus on problem sets and honor code before starting this homework.


2. [11 points] Prove using strong induction that Algorithm 1 correctly outputs the prime factors an integer. The prime factors of \( n \) are a list of primes whose product is \( n \). For example for input 60, the algorithm outputs: “2,2,3,5”, since 2, 3, 5 are all prime, and \( 2 \times 2 \times 3 \times 5 = 60 \). Hint: while proving the inductive step, you should have two cases for the if/else statement.

```plaintext
Input : An integer \( n \) such that \( n \geq 2 \)
Output: String of the prime factors of \( n \)
1  d = 2;
   /* Search for a factor: */
2  while \( n \%d \neq 0 \) do
3      d+ = 1;
4  end
   /* When find a factor: */
5  if \( d == n \) then
6     return “n”;
7  else
8     return Factor(d)+Factor(n/d).
9  end

Algorithm 1: Factor(n)
```

3. The floor and ceiling functions come up often in computer science. Their domain is the real numbers and their codomain is the integers. \( \lfloor x \rfloor \) (“the floor of \( x \)” ) is the largest integer less than or equal to \( x \). \( \lceil x \rceil \) (“the ceiling of \( x \)” ) is the smallest integer greater than or equal to \( x \).

   (a) [2 points] What is \( \lfloor -\sqrt{2} \rfloor \)?
   (b) [3 points] Is the ceiling function surjective? Explain why.
   (c) [3 points] Is the floor function injective? Explain why.
   (d) [11 points] Prove true or prove false: \( \forall x \in \mathbb{R}, \lceil \lfloor x \rfloor \rceil = \lfloor x \rfloor \).
   (e) [11 points] Prove true or prove false: \( \forall x \in \mathbb{R}, 2\lfloor x \rfloor = 2\lfloor x \rfloor \).

4. [11 points] Suppose a procedure involves \( m \) tasks, where task \( i \) can be completed in \( n_i \) ways. Prove using the product rule that there are

\[
\prod_{i=1}^{m} n_i
\]
ways of completing the procedure. Note that
\[ \prod_{i=k}^{j} a_i = a_k \times a_{k+1} \times \cdots \times a_{j-1} \times a_j. \]  \hspace{1cm} (2)

5. [6 points] How many surjective functions are there from set \( A \) to set \( B \) if \( |A| = n \) and \( |B| = 2 \)? Please explain your reasoning. Recall that a function \( f : A \to B \) is surjective iff \( \forall b \in B \exists a \in A, f(a) = b \)

6. How long did you spend on this homework?