CS200 - Problem Set 5

Due: Monday, Oct. 16 to submission server before class

Please read the sections of the syllabus on problem sets and honor code before starting this homework.

1. **[3 points]** Suppose you can prove a statement using induction. Can you also prove the same statement using strong induction? Explain.

2. **[11 points]** Prove using strong induction that Algorithm 1 correctly outputs the prime factors an integer. The prime factors of n are a list of primes whose product is n. For example for input 60, the algorithm outputs: "2,2,3,5", since $2, 3, 5$ are all prime, and $2 \times 2 \times 3 \times 5 = 60$. Hint: while proving the inductive step, you should have two cases for the if/else statement.

```
Input  : An integer n such that n ≥ 2
Output: String of the prime factors of n
d = 2;
/* Search for a factor:                    */
while n%d ≠ 0 do
 |  d+ = 1;
end
/* When find a factor:                     */
if d == n then
 |  return "n";
else
 |  return Factor(d)+Factor(n/d).
end
```

Algorithm 1: Factor(n)

3. The floor and ceiling functions come up often in computer science. Their domain is the real numbers and their codomain is the integers. $\lfloor x \rfloor$ ("the floor of x") is the largest integer less than or equal to x. $\lceil x \rceil$ ("the ceiling of x") is the smallest integer greater than or equal to x.

   (a) **[2 points]** What is $\lfloor -\sqrt{2} \rfloor$?

   (b) **[3 points]** Is the ceiling function surjective? Explain why.

   (c) **[3 points]** Is the floor function injective? Explain why.

   (d) **[11 points]** Prove true or prove false: $\forall x \in \mathbb{R}, \lceil (\lfloor x \rfloor) \rceil = \lfloor x \rfloor$.

   (e) **[11 points]** Prove true or prove false: $\forall x \in \mathbb{R}, \lfloor 2x \rfloor = 2\lfloor x \rfloor$.

4. **[11 points]** Suppose a procedure involves m tasks, where task i can be completed in n_i ways. Prove using the product rule that there are

$$\prod_{i=1}^{m} n_i \tag{1}$$

ways of completing the procedure. Note that

$$\prod_{i=k}^{j} a_i = a_k \times a_{k+1} \times \cdots \times a_{j-1} \times a_j. \tag{2}$$

5. **[6 points]** How many surjective functions are there from set A to set B if $|A| = n$ and $|B| = 2$? Please explain your reasoning. Recall that a function $f : A \to B$ is surjective iff $\forall b \in B \, \exists a \in A, \; f(a) = b$

6. How long did you spend on this homework?