CS200 - Problem Set 4

Due: Monday, Oct. 9 to Canvas before class

Please read the sections of the syllabus on problem sets and honor code before starting this homework.

1. Read section 5.3 of Proof by Richard Hammack. Then read the following poorly written proof of the statement: If n is even, then n^2 is even.

   Proof:

   1. Let $n =$ an integer.
   2. Suppose n is even.
   3. Then $n = 2k$.
   4. $n^2 = (2k)^2$, $(2k)^2 = 4k^2$, so $4k^2 = 2(2k^2)$
   5. Since $(2k^2)$ is an integer, I've shown it is even.

   (The sentences in the proof are numbered to make it easier to reference specific lines in your answer.)

   (a) **[1 point per guideline violation found]** Identify sentences that violate Hammack's mathematical writing guidelines and explain why. (A sentence can violate multiple guidelines, and so can be included multiple times.)

   (b) **[6 points]** Rewrite the proof so that it follows Hammack's mathematical writing guidelines.

2. *Pigeon Hole Principle* **[11 points each]** The pigeonhole principle is an extremely important tool in computer science (see this StackExchange post for just some of its many diverse applications). It states: If you put $n + 1$ pigeons in n cubbies, there must be a cubby with more than one pigeon in it. Create two proofs of this fact, one that uses proof by induction and one that is a contrapositve proof. Each proof should be graded using the standard 11-point scale.

3. *Party-trick Proof* **[11 points]** Suppose you are at a party with 19 acquaintances (so there are 20 people at the party). Prove that there must be at least two people at the party who talked to the same number of people over the course of the evening. (Note: we assume that if Alice talked to Bob, that also means that Bob talked to Alice.)

4. *Fundamental Concept* **[3 points]** When we use a direct proof to prove $P \to Q$ is true, we start by assuming P is true. Why do we not also consider the case that P is false?

5. *Proofs involving Sets*

   (a) **[11 points]** Suppose A and B are sets. Prove $A \subseteq B$ if and only if $A \setminus B = \emptyset$.

   (b) **[11 points]** Suppose A and B are sets. Prove that $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

   (c) **[11 points]** Prove that $\{6n : n \in \mathbb{Z}\} = \{3n : n \in \mathbb{Z}\} \cap \{2n : n \in \mathbb{Z}\}$. Hint: use your result from the previous problem!

6. *Stamps Revisited* **[11 points]** Prove that any postage greater than or equal to 28 cents can be made using five-cent stamps and eight-cent stamps. Hint: try using a proof by cases argument regarding the number of eight-cent stamps as part of the inductive step.

7. How long did you spend on this homework?