**Functions**

- Big-$\Omega$, Big-$\Theta$
- Turning English into math

**Problem with Big-$O$:**
- $x + 1 = O(x^2)$ is correct because big-$O$ is upper bound.
- Need asymptotic lower bound! (Big-$O$ is asymptotic upper bound)

**"big-Omega"**

**def:** Let $f, g : \mathbb{N} \to \mathbb{R}$ then $f(x)$ is $\Omega(g(x))$ if there exist constants $k \in \mathbb{Z}$ and $c \in \mathbb{R}$ such that when $x \geq k$, then

$$f(x) \geq c \cdot g(x)$$

**ex:** $5x + 6 = \Omega(x)$

**Pf:** Note $\forall x \in \mathbb{N}, (x>1) \Rightarrow (6>-x)$. Thus

$$5x + 6 \geq 5x - x \geq 4x$$

So with $k = 1$, $c = 4$, we have $5x + 6 = \Omega(x)$
Big-O = "asymptotic upper bound"
Big-Ω = "asymptotic lower bound"

For linear search, what is a (1) best-case asymptotic lower bound (2) worst-case asymptotic lower bound?

A) Ω(1), O(1)  B) Ω(n), O(1)  C) Ω(1), O(n)
D) Ω(n), O(n)).

In the best-case, the item we are searching for is at beginning of list, so we are done in constant # of steps. Worst case, need to go through the whole list, takes time O(n).

* Asymptotic ≠ worst-case
  - Worst-case asymptotic
  - Best-case asymptotic
  - Average-case asymptotic
Problem with $\Omega$:

\[ X^2 + 1 = \Omega(x) \]

"big Theta"

\[ \text{def: } f(x) = \Theta(g(x)) \iff f(x) = O(g(x)) \land f(x) = \Omega(g(x)) \]

ex: \[ X^2 + 1 = \Theta(x^2) \text{ but } X^2 + 1 \neq \Theta(x), X^2 + 1 \neq \Theta(x^3) \]

Big-$\Theta$: tight asymptotic bound

Squished between upper and lower bounds
Predicates, Quantifiers, & English \rightarrow Math.

\[ P(n) \equiv n \text{ is prime} \]

Predicate is a function! \( P: \mathbb{R} \rightarrow \{\text{true, false}\} \)

\[ n \mapsto \mathbb{Z} \rightarrow \mathbb{N} \]

Not clear what domain is

For a natural number \( n \), \( P(n) \equiv n \text{ is prime} \)
or \( P(n) \equiv \text{the natural number } n \text{ is prime} \).

Now clear, \( P: \mathbb{N} \rightarrow \{\text{true, false}\} \)

Important

- For input to Predicate NO QUANTIFIER
- For any other variable NEED QUANTIFIER

eg. \[ P(n) \equiv \forall \, n : (n \text{ is prime}) \times \]
\[ P(5) \equiv \forall \, 5 : (5 \text{ is prime}) \times \]
\[ P(n) \equiv \exists \, m \in \mathbb{N} : 1 < m < n \wedge m \mid n \]

\[ \quad \text{Need to quantify } m \]
\[ \text{no quantifier for } n. \]
\[ S = \exists \text{ every variable should be quantified.} \]

\[ \forall x, \quad \text{if } x > S \rightarrow x^2 = 10 \]

\[ \exists x : \quad x = 2 \]

For all \( x \) such that \( x > S \), \( T(x) \)

Before: \( \forall x : x > S, T(x) \)
Better: \( \forall x, \quad x > S \rightarrow T(x) \)

\( m \mid n \leftarrow \text{true or false, not } m \mid n = 2 \)

"\( m \) divides \( n \)"

\[ m \mid n \equiv \exists r \in \mathbb{Z} : m \cdot r = n \quad (\text{Domain } n, m \text{ is integers}) \]

\( \forall x_1, x_2 \in \mathbb{R}, \quad \forall x_1 = x_2 \) is always FALSE, because not all \( 2 \) real \( \#\)'s equal each other

Better: \( \forall x_1, x_2 \in \mathbb{R}, \quad x_1 \neq x_2 \rightarrow \ldots \)

We don't have this \( \exists x_1, x_2 \in \mathbb{R}, \quad \forall x_1 = x_2 \) OK!
ex: For $n, m \in \mathbb{N}$

$R(n,m) \equiv$ every natural number less than $m$ divides $n$.

$T(n,m) \equiv$ there is a natural number less than $m$ that divides $n$.

$W(n,m) \equiv n$ and $m$ don't have a common factor

$R(n,m) \equiv \forall p \in \mathbb{N}, p < m \rightarrow p \mid n$

$T(n,m) \equiv \exists p \in \mathbb{N} : p < m \land p \mid n$

$W(n,m) \equiv \forall p \in \mathbb{Z} : p \mid n \land p \mid m$

Rewrite: $\exists x : P(x)$ using $\forall$

- $\neg \forall x, P(x)$ using $\exists$

- $\exists x : P(x) \equiv \forall x, \neg P(x)$

- $\forall x, P(x) \equiv \exists x : \neg P(x)$