Inductive Proof Recipe:

- Set-up (need a predicate $P(n)$)
- Base Case
- Inductive Case (assume $P(k)$)
- Conclusion

Prove: the sum of the first n odd integers equals n^2.

Recall the sum of the first n odd integers is $1 + 3 + \cdots + (2n - 1)$

Set-up

- Let $P(n)$ be the predicate the sum of the first n odd numbers equals n^2. We will prove via induction that $P(n)$ is true for all $n \geq 1$.

Base Case

- Let $P(n)$ be the predicate the sum of the first n odd numbers equals n^2. We will prove via induction that $P(n)$ is true for all $n \geq 1$.
- $P(1)$ is true because the first odd number is $1,$ and $1^2 = 1$.

Inductive Step

Let $k \geq 1$. Assume for induction that $P(k)$ is true. That is, we assume

$$1 + 3 + \cdots + (2k - 1) = k^2$$

The sum of the first $k + 1$ odd numbers is

$$1 + 3 + \cdots + (2k - 1) + (2(k + 1) - 1)$$

Plugging in from the inductive assumption, this is

$$k^2 + 2(k + 1) - 1 = k^2 + 2k + 1 = (k + 1)^\wedge 2$$

Thus $P(k + 1)$ is true.

Conclusion

- Therefore, by induction on n, $P(n)$ is true for all $n \geq 1$.