Math Foundations of Computer Science

Inductive Proof Recipe:

- Let $P(n)$ be the predicate ____________. We will prove, using induction on n, that $P(n)$ is true for all $n \geq$ _
- Base Case: $P(\_)$ is true because ____________
- Inductive Case: Let $k \geq$ _. Assume, for induction, that $P(k)$ is true. That is, we assume ____________. Then we will prove $P(k+1)$ is also true. [a bunch of explanation here]____. Thus, $P(k+1)$ is true.
- Therefore, by induction, $P(n)$ is true for all $n \geq$ _.

Prove: $7^n - 1$ is a multiple of 6 for all integers $n \geq 0$.

(Hint: x is a multiple of 6 if $x = 6 \cdot m$ for an integer m.)

Set-up

- Let $P(n)$ be the predicate $7^n - 1$ is a multiple of 6. We will prove via induction that $P(n)$ is true for all $n \geq 0$.

Base Case

- $P(0)$ is true because $7^0 - 1 = 1 - 1 = 0$, and $0 = 6 \cdot 0$, so 0 is a multiple of 6.

Inductive Case

Let $k \geq 0$. Assume for induction that $P(k)$ is true. That is, we assume $7^k - 1$ is divisible by 6. We will prove $P(k+1)$ is also true. Note

$$7^{k+1} - 1 = (7^k - 1) \cdot 7 + 6.$$

By our inductive assumption, there is an integer m such that $7^k - 1 = 6m$. Plugging in, we have

$$7^{k+1} - 1 = 6m \cdot 7 + 6 = 6(7\mathrm{m} + 1).$$

Since $7m + 1$ is an integer, $P(k+1)$ is true.

Alternative Inductive Case

Let $k \geq 0$. Assume for induction that $P(k)$ is true. That is, we assume $7^k - 1$ is divisible by 6. We will prove $P(k+1)$ is also true. By inductive assumption

$$7^k - 1 = 6\mathrm{m}$$

for some integer m. Multiplying both sides by 7 and then adding 6 to both sides, we have

$$7^{k+1} - 1 = 6m \cdot 7 + 6 = 6(7m + 1)$$

Since m is an integer, $7m + 1$ is an integer, so $P(k+1)$ is true.

Conclusion

- Therefore, by induction on n, $P(n)$ is true for all $n \geq 0$.