Math Foundations of Computer Science
**Inductive Proof Recipe:**

- Let $P(n)$ be the predicate __________. We will prove, using induction on $n$, that $P(n)$ is true for all $n \geq _$
- Base Case: $P(_)$ is true because __________
- Inductive Case: Let $k \geq _. \text{ Assume, for induction, that } P(k) \text{ is true. That is, we assume __________. Then we will prove } P(k + 1) \text{ is also true. [a bunch of explanation here]} ___. \text{ Thus, } P(k + 1) \text{ is true.}
- Therefore, by induction, $P(n)$ is true for all $n \geq _$.

Prove: $7^n - 1$ is a multiple of 6 for all integers $n \geq 0$.

(Hint: $x$ is a multiple of 6 if $x = 6 \cdot m$ for an integer $m$.)
Set-up

- Let $P(n)$ be the predicate $7^n - 1$ is a multiple of 6. We will prove via induction that $P(n)$ is true for all $n \geq 0$. 
Base Case

• $P(0)$ is true because $7^0 - 1 = 1 - 1 = 0$, and $0 = 6 \cdot 0$, so 0 is a multiple of 6.
**Inductive Case**

Let $k \geq 0$. Assume for induction that $P(k)$ is true. That is, we assume $7^k - 1$ is divisible by 6. We will prove $P(k + 1)$ is also true. Note

$$7^{k+1} - 1 = (7^k - 1) \cdot 7 + 6.$$  

By our inductive assumption, there is an integer $m$ such that $7^k - 1 = 6m$. Plugging in, we have

$$7^{k+1} - 1 = 6m \cdot 7 + 6 = 6(7m + 1).$$

Since $7m + 1$ is an integer, $P(k + 1)$ is true.
**Alternative Inductive Case**

Let $k \geq 0$. Assume for induction that $P(k)$ is true. That is, we assume $7^k - 1$ is divisible by 6. We will prove $P(k + 1)$ is also true. By inductive assumption

$$7^k - 1 = 6m$$

for some integer $m$. Multiplying both sides by 7 and then adding 6 to both sides, we have

$$7^{k+1} - 1 = 6m \cdot 7 + 6 = 6(7m + 1)$$

Since $m$ is an integer, $7m + 1$ is an integer, so $P(k + 1)$ is true.
Conclusion

- Therefore, by induction on $n$, $P(n)$ is true for all $n \geq 0$. 