Announcements: Quiz, Equatio, Plickers, Questionnaire

Outline of Course

1. Writing Proofs
   - Induction
   - Constructive
   - Direct
   - Contradiction
   - Contrapositive

We'll start here, to whet your appetite one of the most powerful tools in C.S. tool box

2. Important Math for C.S. (and life!)
   - counting & combinatorics
   - graphs
   - number theory
   - probability
   - growth of functions
   - finite state machines

Motivating Proofs

Q: When you write a program, how do you tell if it works correctly?
   - Try examples
   - Trace variable values
   - Use debugging tools
   - Think logically
   - See if got an "A"

Better approach: Proof: formal method of arguing a statement is true
   "My program outputs correct value"
Induction

Suppose you have unlimited 5¢ stamps and 8¢ stamps. What postage values can you create?

What about 4¢? No!

What about 28¢? What about n¢?

\[ \boxed{55558} \]
Induction: use old solution to get new solution

Suppose

\[ \text{\$} \]  

\[ \begin{array}{c}
\text{\$} \\
\text{\$} \\
\text{\$} \\
\text{\$} \\
\text{\$} \\
\text{\$} \\
\text{\$} \\
\text{\$} \\
\text{add} \\
\text{\$} 15 \\
\text{\$} 16 \\
\end{array} \]  

\[ \text{n \$} \rightarrow (n+1) \text{ \$} \]

Q:

Suppose

\[ \begin{array}{c}
\text{\$} \\
\text{\$} \\
\text{\$} \\
\text{\$} \\
\text{\$} \\
\text{\$} \\
\text{\$} \\
\text{\$} \\
\text{add} \\
\text{\$} 25 \\
\text{\$} 24 \\
\end{array} \]  

\[ \text{n \$} \rightarrow (n+1) \text{ \$} \]

Consequence: If can create \( n \text{ \$} \) with at least 3 \[ \text{\$} \] or at least 3 \[ \text{\$} \], can create \( n+1 \text{ \$} \)
$28^4 = 4 \sqrt[4]{5} + 1 \sqrt[4]{6}$

$29^4 = 1 \sqrt[4]{5} + 3 \sqrt[4]{6}$

$30^4 = 6 \sqrt[4]{5}$

$31^4 = 3 \sqrt[4]{5} + 2 \sqrt[4]{8}$

Q: If $85,693^4 = 5,761 \sqrt[4]{5} + 7111 \sqrt[4]{8}$

then can create $85,694^4$ as

$\frac{5758}{\sqrt[4]{5}} + 7113 \sqrt[4]{8}$

or

$\frac{5766}{\sqrt[4]{5}} + 7108 \sqrt[4]{8}$

A) $5759/7114$

B) $5764/7108$

C) $5766/7108$

D) $5758/7113$

* Any postage $\geq 28^4$ is possible

Start at $28^4 \rightarrow 29^4 \rightarrow 30^4 \rightarrow \ldots \ 85,693^4 \rightarrow \ldots$

find first solution, & the rest fall into place
Principle of induction: solution to problem of size n
gives solution to problem of size n+1.

Needed to have previous solution to get next solution!
Once you get 2nd solution, we're good.

Inductive Metaphors

Ladder

1. Show how to get on first rung of ladder
2. Show how to move from one rung to next

Formal Inductive Proof

Proofs have a unique style/language
- Essay vs. Texting vs News article vs. lab notebook

All unique styles

This class → proof language.

Induction proof has a recipe, so a bit easier to start with than other proofs
Inductive proof recipe:

(Set-up)
Let $P(n)$ be the predicate $n$ can be formed from 5¢ and 8¢ stamps.

We will prove, using induction on $n$, that $P(n)$ is true for all $n \geq 28$.

(Base-case)
Base case: $P(28)$ is true because

(Inductive Case)
Inductive case: Let $k \geq 28$. Assume, for induction, that $P(k)$ is true. That is, we assume we can make $k$¢ postage out of 5¢ and 8¢ stamps. Then we will prove $P(k+1)$ is also true.

First note

Then

Plugging in

Thus $P(k+1)$ is true

(Conclusion)
Therefore, by induction, $P(n)$ is true for all $n \geq 28$. 

Each sentence logically follows from previous
Notice:
- Complete sentences or equations
- "we"
- "Let" in setup

Q: Write inductive proof that \( 7^n - 1 \) is a multiple of 6 for all integers \( n \geq 0 \).

(Hint: \( x \) is a multiple of 6 if \( x = 6 \cdot m \) for some integer \( m \).)

\[ \text{Let } P(n) \text{ be the predicate } \ 7^n - 1 \text{ is a multiple of 6. We will prove via induction that } P(n) \text{ is true for all } n \geq 0. \]

\[ \text{Base Case: } P(0) \text{ is the statement } 7^0 - 1 = 0 = 0 \cdot 6, \text{ so the statement is true.} \]

\[ \text{Inductive Case: Let } k \geq 0. \text{ Assume, for induction, that } P(k) \text{ is true. That is, we assume } 7^k - 1 \text{ is a multiple of 6.} \]

This means \( 7^k - 1 = 6 \cdot m \) for some integer \( m \). We will prove \( P(k+1) \) is true. Note

\[ 7^{k+1} - 1 = (7^k - 1) \cdot 7 + 7 - 1 = (7^k - 1) \cdot 7 + 6 \]

By our inductive assumption, \( 7^k - 1 = 6 \cdot m \). Thus
\[ 7^{k+1} - 1 = 6m \cdot 7 + 6 = 6(7 \cdot m + 1), \]
which is a multiple of 6. Thus $P(k)$ is true.

Alternative Inductive case:

Inductive Case: Let $k \geq 0$. Assume for induction, that $P(k)$ is true. That is, we assume $7^k - 1$ is a multiple of 6. This means $7^k - 1 = 6 \cdot m$ for some integer $m$. We will prove $P(k+1)$ is true. If we multiply both sides by 7, we have

\[ (7^k - 1) \cdot 7 = 7^{k+1} - 7 = 6 \cdot 7 \cdot m \]

Then adding 6 to both sides, we have

\[ 7^{k+1} - 1 = 6 \cdot 7 \cdot m + 6 = 6(7m + 1), \]

Thus $7^{k+1} - 1$ is a multiple of 6, and $P(k+1)$ is true.

Therefore, by induction on $n$, $P(n)$ is true for all $n \geq 0$. 

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