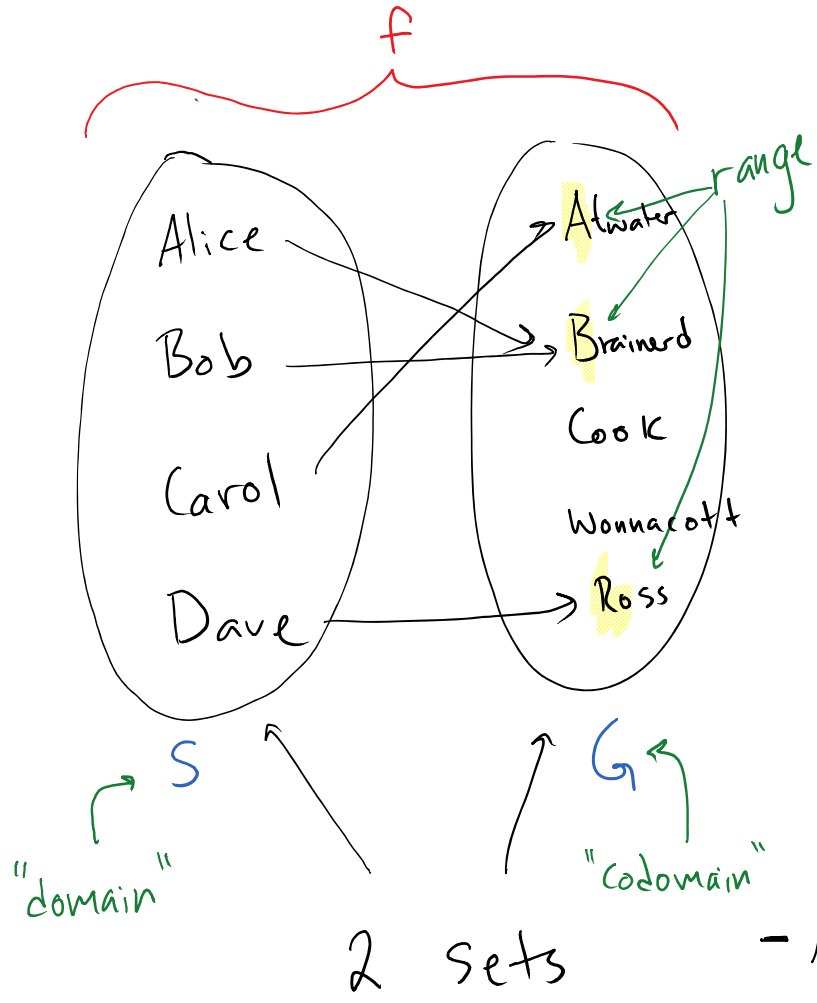


# Functions

All this notation is important because when we write about functions in proofs, need to have accurate words

ex: Common affiliation



You give the function a student name as input, it gives a grade as output

We write:

•  $f: S \rightarrow G$

means "f is a function from domain S to codomain G"

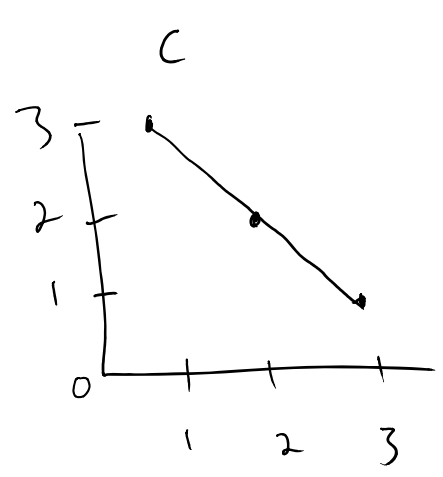
•  $f(\text{Carol}) = \text{Atwater}$

- Atwater is "image" of carol
- Carol is "preimage" of Atwater

ex: Let  $f: \{1,2,3\} \rightarrow \{1,2,3\}$

A  
 $1 \rightarrow 3$   
 $2 \rightarrow 2$   
 $3 \rightarrow 1$

B  
 $f(x) = -x + 4$



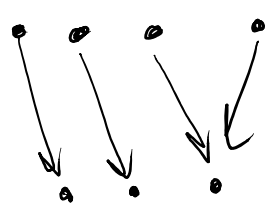
D

X	1	2	3
f(x)	3	2	1

Which is an incorrect representation?

2 important properties

Surjection

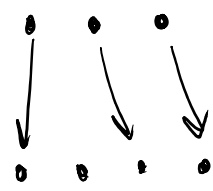


Surjective

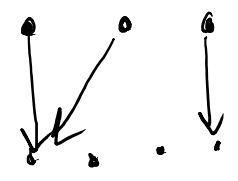


Not Surjective

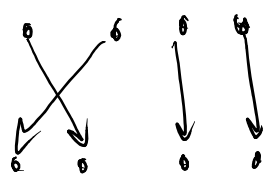
Injection



Injective



Not injective



Injective & Surjective

Surjective: every codomain element has a preimage

Injective: no two elements of domain map to same element of co-domain

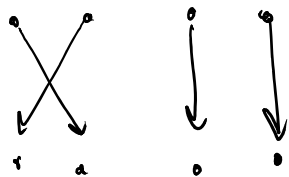
Q: Write using math:

• A function  $f: S \rightarrow G$  is surjective if  $\forall x \in G, \exists y \in S: f(y) = x$

• A function  $f: S \rightarrow G$  is injective if  $\sim \exists y_1, y_2 \in S: (y_1 = y_2) \wedge f(y_1) = f(y_2)$

One-to-One = Injective

Bijective (both surjective and injective)



Brainstorm examples of each in real life.  
Try to express using math notation

Only  
Surjective

$$f: \{p: p \text{ is a person}\} \rightarrow \{1, 2, \dots, 12\}$$

$f(p)$  is month person  
 $p$  was born in

Only  
Injective

$$f: \{p: \text{student at Middlebury}\} \rightarrow \{s: s \text{ is a Middlebury e-mail}\}$$

Bijective  
(both injective & surjective)

$$f: \{r: \text{single in coffin}\} \rightarrow \{p: \text{person in single in coffin}\}$$

Nothing

$$f: \{p: p \text{ is a person}\} \rightarrow \mathbb{N}$$

$f(p)$  is age of person  $p$