Shape from Shading

Concept
The shading of an image creates a certain perception of it’s shape:

The idea of Shape from Shading is to look at how an image is shaded and reconstruct its shape.

Process
1. Input the intensity of the pixel
2. Get the direction of the surface normal as Output
3. Reconstruct shape from surface normals

Assumptions
- Orthographic Projection (Estimate of perspective proj. if object is far from camera and close to optical axis)
- Light source is far away (So position of image doesn’t alter shading)
- Secondary illumination is insignificant (only light from main source illuminates object)
- No cast shadows in image (No part of the object blocks light from another part)

Reflectance
- The intensity (Irradiance) of a pixel is a function of how much light that point on the object is reflecting.
- $Irrad(x, y) = Ref(p(x,y), q(x,y))$. This Ref() function is the Reflectance.
- The Reflectance is a function of:
  a) The Direction of the Surface Normal $[p(x,y), q(x,y)]$
  b) The direction from which the light is coming (camera position)
  c) The direction in which the light is being reflected
  d) The Albedo (ratio of incoming radiation to outgoing radiation)
- If we assume that the object is Lambertian and that albedo of the object is constant throughout the object, then we can eliminate properties c) and d) respectively.
- So the Reflectance is only a function of the surface normal and light source directions.
- It turns out that $Irrad(x, y) = Ref(p, q) = \cos(i)$ where $i$ is the angle between the light direction vector and the surface normal vector.
- So now we can look at the intensity of each pixel and get the direction of the surface normal because we already know that direction of light.
- We can then construct a Reflectance Map for any given direction of light:
Shape Reconstruction

- Now that we have a Reflectance Map, for each pixel we can:
  - Look at the intensity
  - Compare intensity to Reflectance Map
  - Obtain a set of possible surface normal directions for that pixel
- Now we need to reconstruct the shape:

1. One way of doing that is by using **Gradients** - Matching the gradient of intensity change in the image with the gradient of intensity change in the reflectance map:

   ![Gradients Image]

2. Another way is through **Photometric Stereo** - Using two different lighting conditions to obtain two different Reflectance Maps and then superimposing them. So we can check a pixel under each lighting condition and look at the intersection of the two curves corresponding to each map:

   ![Photometric Stereo Image]

3. Another interesting concept is **Helmholtz Stereo**
   - This isn’t really shape from shading but it’s a method of stereo vision
   - The idea is that if you interchange the positions of the camera and the light source, looking at a specific pixel, that pixel will have the same intensity in both images.

   ![Helmholtz Stereo Image]

   - We can therefore perform stereo vision to reconstruct the shape, without the intensities changing between images.
Sources

- Nalwa, Vishvjit S., A Guided Tour of Computer Vision.
- http://www.eecs.harvard.edu/~zickler/helmholtz.html