Analysis:
We ended up choosing our optimal number of centroids to be $k = 6$. This was inferred through the elbow method heuristic. We note that as $k$ increases, the cost decreases. This makes sense as we are fitting our model closer and closer to the training data. To prevent overfitting the model, the elbow method suggests choosing $k$ when the rate at which the error decreases slows down. While this trend seen in the graph is not perfect for the elbow method (it is quite curved and doesn’t change suddenly), we believe that combining this heuristic and subsequent in-depth looks at our clusters confirms $k = 6$.

One important first step in analyzing the clusters for our algorithm involved comparing the cluster centers and the average across all countries for all features.
Considering these two graphs, we note how:

- Cluster 1 is large for features 1, 4, 6, 7 and 10
- Cluster 2 is normally around or below the average except for features 3, 5, 12 and 16
- Cluster 3 is often around or below the average except for features 2, 5, 6 and 16, where it passes the average by a large margin
- Cluster 4 is mostly near the average except for features 8 and 9, where it is slightly above.
- Cluster 5 is very often significantly higher than the average: for features 1, 4, 10, 11, 13, 14 and 15
- Cluster 6 is again near the average for most features, and slightly above for features 11 and 12

Additionally, we also made sure clustering was somewhat evenly balanced. This turned out to hold somewhat true as the clusters were the following sizes:

- Cluster 1: 36
- Cluster 2: 19
- Cluster 3: 48
- Cluster 4: 18
- Cluster 5: 28
- Cluster 6: 45

While there is definitely some variation between cluster sizes, this is expected, especially considering the varying number of countries in different continents / of different economies.

*Example: Africa vs North America*

**CLUSTER 1**
- Argentina
- Barbados
- Bolivia
- Central African Republic
- Chile
- Colombia
- Costa Rica
- Croatia
- Ecuador
- El Salvador
- Estonia
- Guatemala

**CLUSTER 2**
- Bhutan
- Bosnia and Herzegovina
- Cote d'Ivoire
- Dominican Republic

---

<table>
<thead>
<tr>
<th>Cluster 1</th>
<th>Cluster 2</th>
<th>Additional countries</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>Guinea-Bissau</td>
<td>Saint Lucia</td>
</tr>
<tr>
<td>Barbados</td>
<td>Hungary</td>
<td>Samoa</td>
</tr>
<tr>
<td>Bolivia</td>
<td>Ireland</td>
<td>Sao Tome and Principe</td>
</tr>
<tr>
<td>Central African Republic</td>
<td>Israel</td>
<td>Senegal</td>
</tr>
<tr>
<td>Chile</td>
<td>Italy</td>
<td>Seychelles</td>
</tr>
<tr>
<td>Colombia</td>
<td>Japan</td>
<td>Singapore</td>
</tr>
<tr>
<td>Costa Rica</td>
<td>Malta</td>
<td>Slovenia</td>
</tr>
<tr>
<td>Croatia</td>
<td>Marshall Islands</td>
<td>Solomon Islands</td>
</tr>
<tr>
<td>Ecuador</td>
<td>Mauritius</td>
<td>South Sudan</td>
</tr>
<tr>
<td>El Salvador</td>
<td>Niger</td>
<td>Spain</td>
</tr>
<tr>
<td>Estonia</td>
<td>Panama</td>
<td>Uruguay</td>
</tr>
<tr>
<td>Guatemala</td>
<td>Portugal</td>
<td>Vanuatu</td>
</tr>
<tr>
<td>Bhutan</td>
<td>Gabon</td>
<td>Lesotho</td>
</tr>
<tr>
<td>Bosnia and Herzegovina</td>
<td>Ghana</td>
<td>Liberia</td>
</tr>
<tr>
<td>Cote d'Ivoire</td>
<td>Haiti</td>
<td>Malawi</td>
</tr>
<tr>
<td>Dominican Republic</td>
<td>Kiribati</td>
<td>Mali</td>
</tr>
<tr>
<td>Country 1</td>
<td>Country 2</td>
<td>Country 3</td>
</tr>
<tr>
<td>---------------------------</td>
<td>------------------------</td>
<td>----------------------------------</td>
</tr>
<tr>
<td>Afghanistan</td>
<td>Jamaica</td>
<td>South Africa</td>
</tr>
<tr>
<td>Algeria</td>
<td>Kuwait</td>
<td>Swaziland</td>
</tr>
<tr>
<td>Bahrain</td>
<td>Kyrgyzstan</td>
<td>Syrian Arab Republic</td>
</tr>
<tr>
<td>Bangladesh</td>
<td>Libya</td>
<td>Tajikistan</td>
</tr>
<tr>
<td>Botswana</td>
<td>Maldives</td>
<td>Timor-Leste</td>
</tr>
<tr>
<td>Burkina Faso</td>
<td>Mauritania</td>
<td>Togo</td>
</tr>
<tr>
<td>Cape Verde</td>
<td>Mexico</td>
<td>Tunisia</td>
</tr>
<tr>
<td>Congo</td>
<td>Morocco</td>
<td>Tuvalu</td>
</tr>
<tr>
<td>Democratic Republic of</td>
<td>Mozambique</td>
<td>Uganda</td>
</tr>
<tr>
<td>the Congo</td>
<td>Namibia</td>
<td>United Arab Emirates</td>
</tr>
<tr>
<td>Egypt</td>
<td>Nepal</td>
<td>United Republic of</td>
</tr>
<tr>
<td>Fiji</td>
<td>Nicaragua</td>
<td>Tanzania</td>
</tr>
<tr>
<td>Gambia</td>
<td>Oman</td>
<td>Uzbekistan</td>
</tr>
<tr>
<td>Georgia</td>
<td>Pakistan</td>
<td>Yemen</td>
</tr>
<tr>
<td>Grenada</td>
<td>Qatar</td>
<td>Zimbabwe</td>
</tr>
<tr>
<td>India</td>
<td>Saudi Arabia</td>
<td>Palestine (State of)</td>
</tr>
<tr>
<td>Iraq</td>
<td>Somalia</td>
<td></td>
</tr>
<tr>
<td>Nauru</td>
<td>Philippines</td>
<td>Zambia</td>
</tr>
<tr>
<td>Nigeria</td>
<td>Sierra Leone</td>
<td></td>
</tr>
<tr>
<td>Paraguay</td>
<td>Sudan</td>
<td></td>
</tr>
</tbody>
</table>

**Cluster 4**

<table>
<thead>
<tr>
<th>Country 1</th>
<th>Country 2</th>
<th>Country 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>Iceland</td>
<td>Sweden</td>
</tr>
<tr>
<td>Belgium</td>
<td>Liechtenstein</td>
<td>Switzerland</td>
</tr>
<tr>
<td>Canada</td>
<td>Luxembourg</td>
<td>United Kingdom of Great Britain</td>
</tr>
<tr>
<td>Denmark</td>
<td>Netherlands</td>
<td>Britain and Northern Ireland</td>
</tr>
<tr>
<td>Finland</td>
<td>New Zealand</td>
<td>Ireland</td>
</tr>
<tr>
<td>France</td>
<td>Norway</td>
<td>United States of America</td>
</tr>
<tr>
<td>Germany</td>
<td>Palau</td>
<td></td>
</tr>
</tbody>
</table>

**Cluster 5**

<table>
<thead>
<tr>
<th>Country 1</th>
<th>Country 2</th>
<th>Country 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Andorra</td>
<td>China</td>
<td>Jordan</td>
</tr>
<tr>
<td>Angola</td>
<td>Comoros</td>
<td>Lao People's Democratic Republic</td>
</tr>
<tr>
<td>Antigua and Barbuda</td>
<td>Cuba</td>
<td>Republic</td>
</tr>
<tr>
<td>Australia</td>
<td>Czech Republic</td>
<td>Monaco</td>
</tr>
<tr>
<td>Azerbaijan</td>
<td>Djibouti</td>
<td>Mongolia</td>
</tr>
<tr>
<td>Bahamas</td>
<td>Equatorial Guinea</td>
<td>Rwanda</td>
</tr>
<tr>
<td>Benin</td>
<td>Eritrea</td>
<td>San Marino</td>
</tr>
<tr>
<td>Burundi</td>
<td>Ethiopia</td>
<td>Sri Lanka</td>
</tr>
<tr>
<td>Cameroon</td>
<td>Guinea</td>
<td>Thailand</td>
</tr>
<tr>
<td>Chad</td>
<td>Iran (Islamic Republic of)</td>
<td></td>
</tr>
</tbody>
</table>
### CLUSTER 6

<table>
<thead>
<tr>
<th>Albania</th>
<th>Kenya</th>
<th>Saint Kitts and Nevis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Armenia</td>
<td>Latvia</td>
<td>Saint Vincent and the Grenadines</td>
</tr>
<tr>
<td>Belarus</td>
<td>Lebanon</td>
<td>Serbia</td>
</tr>
<tr>
<td>Belize</td>
<td>Lithuania</td>
<td>Slovakia</td>
</tr>
<tr>
<td>Brazil</td>
<td>Madagascar</td>
<td>Brunei Darussalam</td>
</tr>
<tr>
<td>Brunei Darussalam</td>
<td>Malaysia</td>
<td>Bolivia</td>
</tr>
<tr>
<td>Bulgaria</td>
<td>Micronesia (Federated States of)</td>
<td>Cambodia</td>
</tr>
<tr>
<td>Cambodia</td>
<td>Myanmar</td>
<td>Czech Republic</td>
</tr>
<tr>
<td>Cyprus</td>
<td>Montenegro</td>
<td>Dominica</td>
</tr>
<tr>
<td>Democratic People's Republic of Korea</td>
<td>Papua New Guinea</td>
<td>Dominican Republic</td>
</tr>
<tr>
<td>Greece</td>
<td>Peru</td>
<td>Greece</td>
</tr>
<tr>
<td>Guyana</td>
<td>Republic of Korea</td>
<td>Honduras</td>
</tr>
<tr>
<td>Honduras</td>
<td>Republic of Moldova</td>
<td>Indonesia</td>
</tr>
<tr>
<td>Indonesia</td>
<td>Romania</td>
<td>Kazakhstan</td>
</tr>
<tr>
<td>Kazakhstan</td>
<td>Russian Federation</td>
<td></td>
</tr>
</tbody>
</table>

The countries in these clusters seem to be related to each other, for example cluster 4 consists mainly of economic powerhouses in North America and Europe. Finally, we decided to look in depth at Peru, the country where Luke grew up in.

Peru was placed in cluster number 6 and we expected it to be surrounded by other Latin American countries. Surprisingly, this was not really the case, as only Venezuela and Guyana are included in this cluster. It mainly contains former Soviet countries. One thing that we did notice across most of these countries is that they tend to be (economically) developing nations, many of which have had political difficulties in recent memory. Despite this trend, there are some countries that are unexpectedly in this cluster such as Saint Vincent and the Grenadines, a small island-nation in the Caribbean.