

Lecture 23: Sorting and Complexity

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CSCI 101
Spring 2018

Today

- Announcements
 - Thursday: Bring computer for evaluations
 - Final exam: self-scheduled; 2 sheets of notes allowed
- Computational Complexity
 - Big-O notation to describe # of operations
 - Last week: complexity of search algorithms
- Sorting Algorithms
 - Elementary methods are $O(n^2)$
 - Divide-and-Conquer methods are $O(n \log n)$

Big-O notation

- Use a function to describe number of basic operations in terms of **input size**
- The function includes only the dominant terms, ignoring constants
- Example: list traversal is $O(n)$ for a list of n values

Binary Search

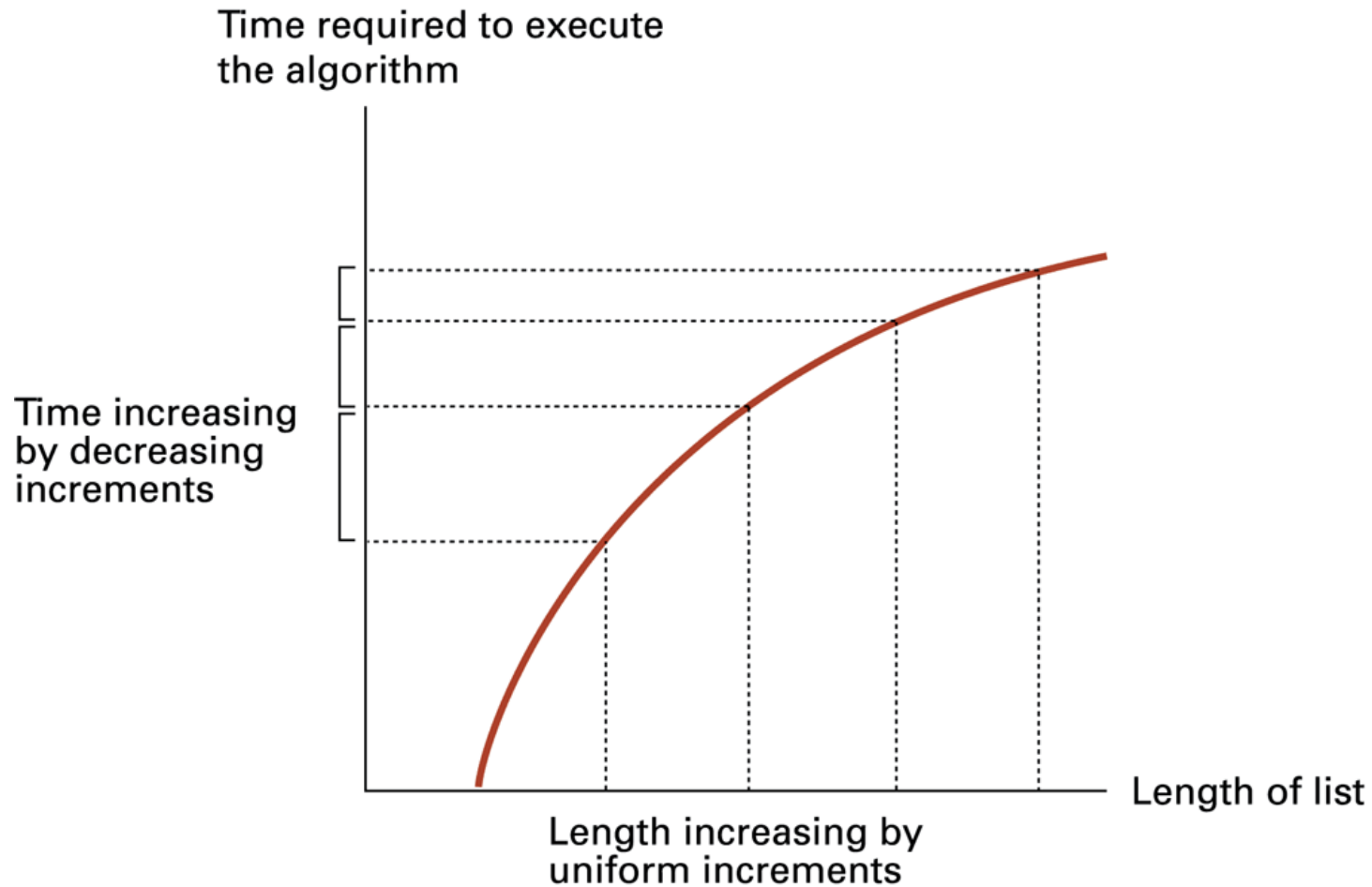
We can search **faster** if the list is **sorted**

→ Compare middle element to the target, then refine search to one half of list

2	5	8	11	15	16	21	24	29	41	45	58	71	85	92	95
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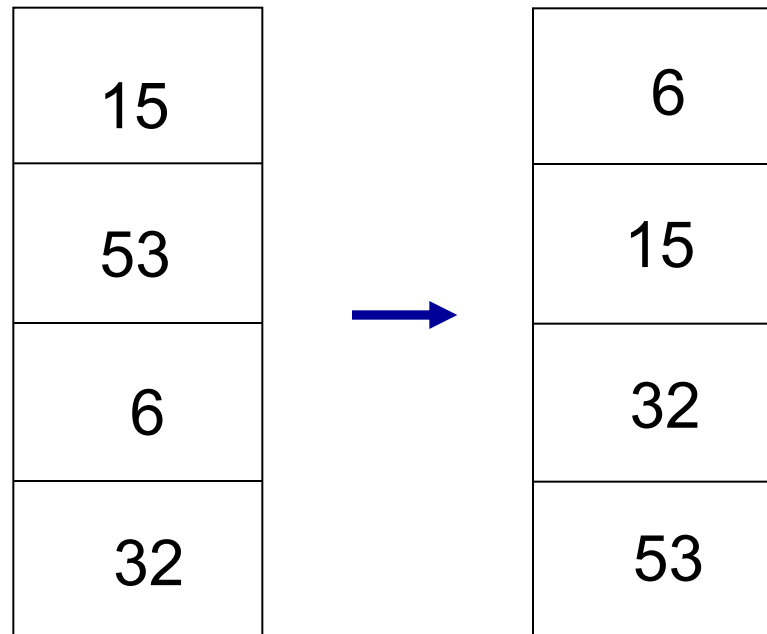
Number of operations for a list of n elements: $O(\log_2 n)$ or $O(\log n)$

Binary Search: $O(\log n)$



Sorting

How to sort n values into increasing order?



Sorting Algorithms

Selection Sort $\longrightarrow O(n^2)$

Find smallest value and **swap** into first position. Repeatedly **select** the smallest $n-1$ times.

Insertion Sort $\longrightarrow O(n^2)$

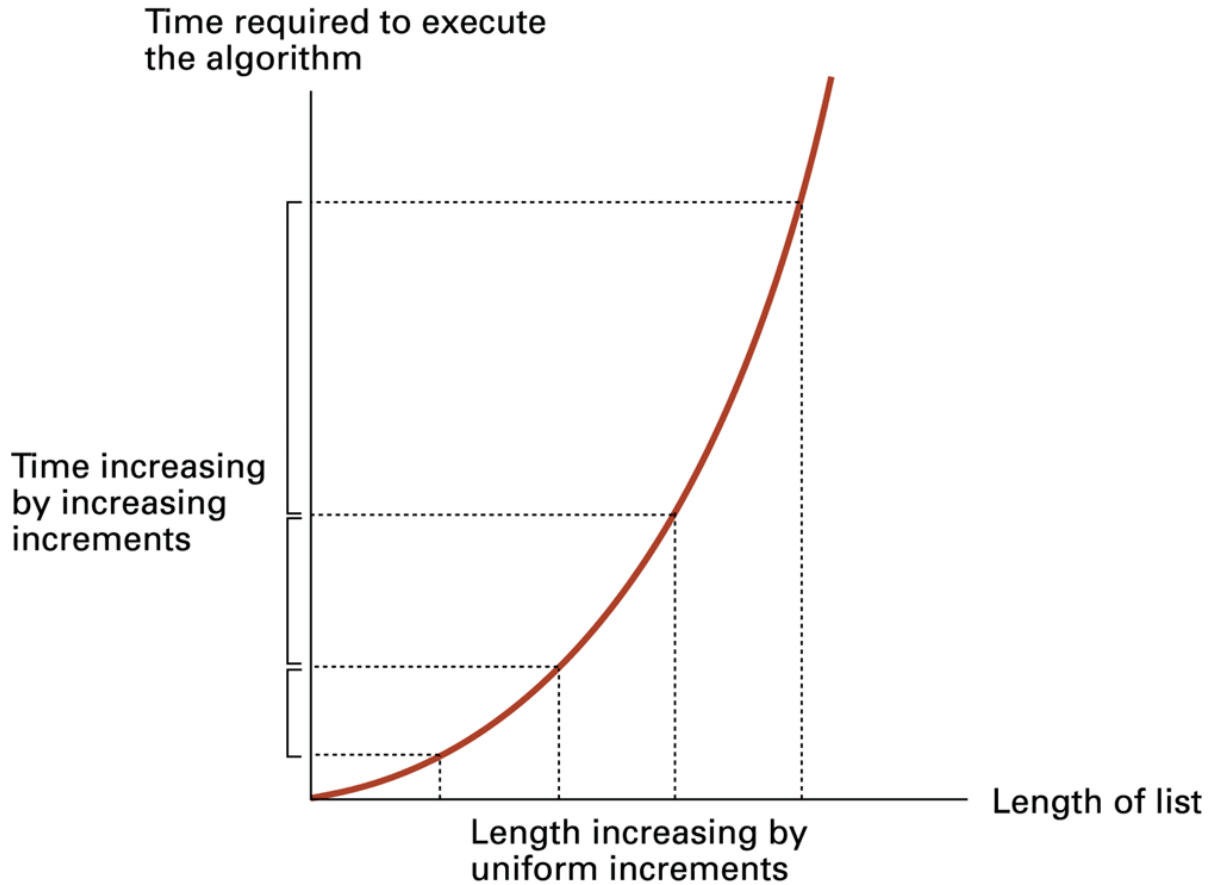
Examine each element in turn, **inserting** it into its proper position among the **already sorted** values to the left.

Bubble Sort $\longrightarrow O(n^2)$

Swap **neighboring** values if out of order (largest **bubbles** to end). Do this $n-1$ times.

Insertion Sort is $O(n^2)$

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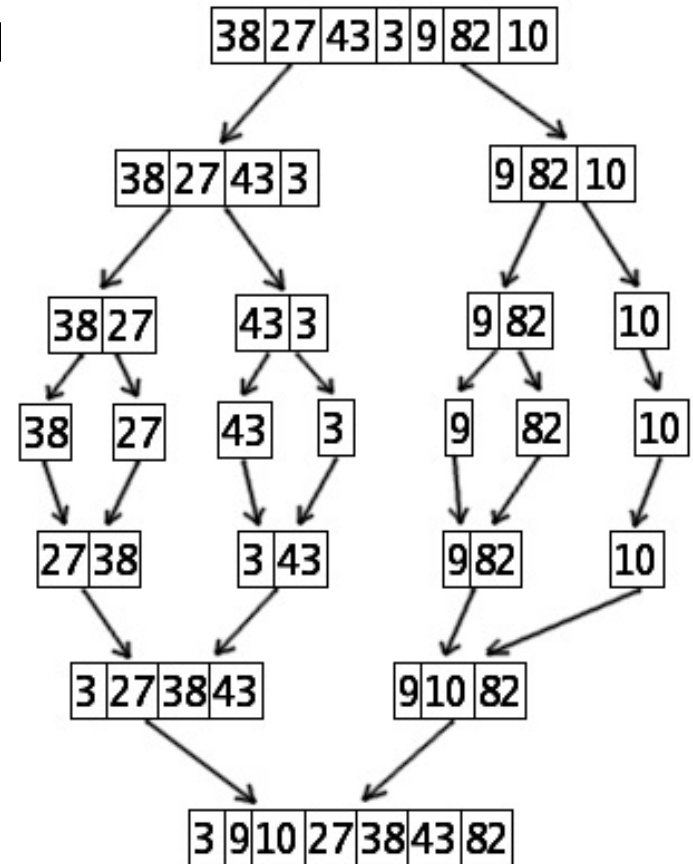


Merge Sort

If a list has only 1 item, then we're done

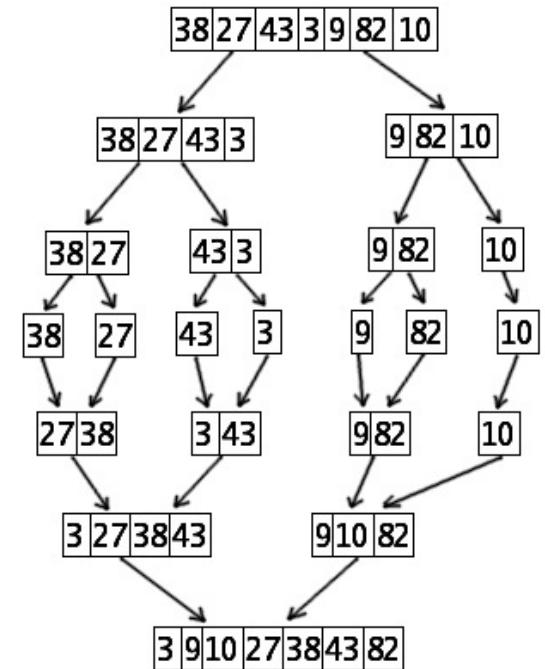
Else:

1. Split the list in half
2. Recursive sort each half
3. Merge the two sorted halves together (how?)



Merge Sort Complexity

- How many **levels of splits**?
- How many **basic operations at each level**?
 - Hint: how many basic operations to merge two lists of size $n/2$?
 - Hint: how many basic operations to merge four lists of size $n/4$?
- Merge sort complexity is $O(\log n) \times O(n) = O(n \log n)$



Summary

- **Computational complexity** describes how the time needed to execute an algorithm increases as its input size increases
- **Big-O notation** summarizes an algorithm's complexity in terms of its input size
 - Linear search: $O(n)$
 - Selection sort: $O(n^2)$
 - Bubble sort: $O(n^2)$
 - Binary search: $O(\log n)$
 - Insertion sort: $O(n^2)$
 - Merge sort: $O(n \log n)$