CS 461: Problem Set One

Due: Wed 2015-03-04

1. [7 points] In order to display our OpenGL world, the graphics pipeline needs to figure out what point in our world each pixel in the canvas corresponds to. You have already had to figure this out to implement clicking. I would like you to formalize this. We have the view (canvas), which has an origin in the upper left, \( y \) increasing down, a width of \( V_W \), and a height of \( V_H \). Our world (OpenGL-land) has its origin in the center, a width of \( W_w \) and a height of \( W_H \).

   ![View and World Diagram]

   a. Write a pair of formulas to convert a point in view space \((x_v, y_v)\) to the appropriate point in world space \((x_w, y_w)\).

   b. Construct a transformation matrix to perform this calculation (work in 2D).

2. [5 points] We can create the inverse of a rotation by using the transpose of the rotation matrix. Use matrix multiplication and a little trigonometry to demonstrate that this is true.

3. [5 points] Describe in words what this 2D transformation matrix does

\[
\begin{pmatrix}
0 & -1 & 1 \\
1 & 0 & 1 \\
0 & 0 & 1
\end{pmatrix}
\]

4. [8 points] Prove that the following matrix operations are commutative

   1. two translations
   2. two scalings

5. [5 points] Prove that rotation in 2D is additive by demonstrating that the concatenation of two rotations \( R(\theta) \) and \( R(\phi) \) is equivalent to a single rotation \( R(\theta + \phi) \).

You may find that you want to use some trig identities to solve some of these. A little googling will find some, but this is a good condensed set of them: [http://www.mathwords.com/t/trig_identities.htm](http://www.mathwords.com/t/trig_identities.htm)