1. Read Kozen Lectures 34–35. See the posted Reduction examples.

2. Tell whether the following problems are decidable or undecidable. Give a short justification for each (formal reduction not required).

(a) whether $L(M)$ is r.e., where $M$ is a Turing machine
(b) whether $L(M) \cap \overline{L(M)} = \emptyset$, where $M$ is a Turing machine
(c) whether $L(M) = L(M)^R$, where $M$ is a Turing machine and $L(M)^R$ is the reverse of all the strings in $L(M)$.

3. Prove that the set of Turing machines

$$TOTAL = \{M \mid M \text{ halts on all inputs}\}$$

is not r.e.

Hint: Recall that $HP = \{M \# x \mid M \text{ halts on } x\}$ is r.e. but not recursive, so therefore $\overline{HP}$ is not r.e. Show $\overline{HP}$ reduces to $TOTAL$, i.e., describe a new machine $N = \sigma(M\#x)$ such that $N$ halts on all inputs iff $M$ does not halt on $x$.

4. Prove that the sets $A = \{M \mid M \text{ does not halt on } \epsilon\}$ and $B = \{M \mid L(M) = \emptyset\}$ are not r.e.

Hint: Reduce $\overline{HP}$ to one of $A$ or $B$, then reduce that set to the other one.

5. Prove that the set $B = \{M \mid M \text{ accepts at least 301 strings}\}$ is r.e. but not co-r.e. (that is, show $B$ is an r.e. set but its complement $\overline{B}$ is not r.e.).

6. Recall the linear bounded automata (LBAs) you defined formally as part of Homework 8. The LBAs are a restricted class of one-tape Turing machines that are not allowed to write on the tape outside of the input area. The input string is enclosed between left and right endmarkers, $\upuparrows$ and $\downdownarrows$, and the machine is constrained never to move to the left of the $\upuparrows$ nor to the right of the $\downdownarrows$. It can read and write all it wants between the endmarkers. Recall that on Homework 8 you proved the halting problem for LBAs is decidable.

(a) Prove by diagonalization that there exists a recursive set that is not accepted by any LBA.

(b) Prove that the emptiness problem for LBAs (given an LBA $M$, is $L(M) = \emptyset$?) is undecidable.