Sample Proofs by Induction

1. See Rosen Section 5.1.

2. Adapted from How to Prove It, by Daniel Velleman

Prove that, for \( n \geq 3 \), if \( n \) distinct points on a circle are connected in consecutive order with straight lines, then the interior angles of the resulting polygon add up to \((n - 2)180^\circ\).

**Proof:**

By induction on \( n \). \( P(n) \) is the proposition that if \( n \) distinct points on a circle are connected in consecutive order with straight lines, then the interior angles of the resulting polygon add up to \((n - 2)180^\circ\).

Base case. \( P(3) \): If 3 distinct points on a circle are connected with straight lines, then the resulting polygon is a triangle, and we know that the interior angles of a triangle add up to \(180^\circ\).

Inductive step: Show that \( P(k)P(k + 1) \). We assume the inductive hypothesis, \( P(k) \) and must show that \( P(k + 1) \) follows from it. To be explicit, \( P(k + 1) \) is the proposition that if \( k + 1 \) distinct points on a circle are connected in consecutive order with straight lines, then the interior angles of the resulting polygon add up to \(((k + 1) - 2)180^\circ\).

Consider the polygon \( P \) formed by connecting some \( k+1 \) distinct points \( A_1, A_2, \ldots, A_{k+1} \) on a circle. If we skip the last point \( A_{k+1} \), then we get a polygon \( P' \) with only \( k \) vertices, and by the induction hypothesis the interior angles of this polygon add up to \((k - 2)180^\circ\). Now, the sum of the interior angles of \( P \) is equal to the sum of the interior angles of \( P' \) plus the sum of the interior angles of the triangle \( A_1A_kA_{k+1} \). Since the sum of the interior angles of the triangle is \(180^\circ\), we can conclude that the sum of the interior angles of \( P \) is

\[
(k - 2)180^\circ + 180^\circ = ((k + 1) - 2)180^\circ,
\]

as required.

Having shown both the base case and inductive step, we have proven the claim.