Warm-up Problems

1. Rewrite each of the following statements in English in the form \( p \rightarrow q \). For example, the statement “I catch cold if I eat ice cream” should be rewritten “I eat ice cream \( \rightarrow \) I catch cold.”

   (a) Winds from the south imply a strong thaw.
   (b) Wily gets caught whenever he cheats.
   (c) You can access the website only if you pay the subscription fee.

2. If \( S \) is the statement \( p \rightarrow q \), the **converse**, **inverse**, and **contrapositive** of \( S \) are defined to be the statements \( q \rightarrow p \), \( \neg p \rightarrow \neg q \), and \( \neg q \rightarrow \neg p \), respectively.

   (a) *I open my umbrella whenever it rains.*
      Rewrite the above sentence in the form \( p \rightarrow q \). Then write a natural sounding English sentence that represents its inverse.
   (b) *I miss class only if I am unwell.*
      Rewrite the above sentence in the form \( p \rightarrow q \). Then write a natural sounding English sentence that represents its contrapositive.
   (c) *You can’t invent unless you are curious and knowledgeable.*
      Rewrite the above sentence in the form \( p \rightarrow q \), using the symbol \( \neg \) wherever necessary. Then write a natural sounding English sentence that represents its converse.

3. **Practice with quantifiers**

   (a) Let \( \psi \equiv \exists x \in S (P(x) \land \forall y \in S ((y \neq x) \rightarrow \neg P(y))) \), where \( S = \{a, b, c, d, e\} \).
      i. Specify a predicate \( P : S \rightarrow \{T, F\} \) for which \( \psi \) is true.
      ii. Specify a predicate \( P : S \rightarrow \{T, F\} \) for which \( \psi \) is false.

   (b) Let \( S \) be an infinite set of people and \( A \) be an infinite set of tasks. Let \( P(x, y) \) be a predicate that states that person \( x \) can perform task \( y \). Translate the following English sentence into a statement involving only logical operations, quantifiers, variables, and \( S, A, \) and \( P \).
      “No matter which person you consider, there is someone else who can perform all the tasks that the former can and more.”

   (c) Write down the negation of the above logical statement, pushing the negation as deeply as possible using Demorgan’s laws.

   (d) Let \( p(x) \) state “\( x \) is prime,” \( q(x) \) state “\( x \) is even,” and \( r(x, y) \) state “\( x = y \).” Using these predicates, boolean operations, and quantifiers, rewrite the statement “There is one and only one even prime.”