Assembly Line Problem

- Cars produced in factory with 2 assembly lines.
- Each line has \( n \) stations (that add car parts).
- Stations \( S_{1,j} \) and \( S_{2,j} \) for \( j = 1, \ldots, n \).
- Stations \( S_{1,j} \) and \( S_{2,j} \) perform same task but possibly at different speeds.
  - \( a_{1,j} \) = time required at \( S_{1,j} \)
  - \( a_{2,j} \) = time required at \( S_{2,j} \)

Cars usually stay on one line (time from \( S_{1,i} \) to \( S_{1,i+1} \) + \( S_{2,i} \) to \( S_{2,i+1} \) negligible).

"Special" Rush order cars can switch from line 1 to line 2 to speed up.

- \( t_{1,j} \) = time to switch from \( S_{1,j} \) to \( S_{2,j+1} \)
- \( t_{2,j} \) = time to switch from \( S_{2,j} \) to \( S_{1,j+1} \)
Goal:
Given values for \(a_{ij}, a_{kj}, t_{pj}, t_{kj}\), find set of stations to visit that will minimize overall time

Optimization problem: goal is to minimize or maximize a specified value.

Other optimization problems: coin changing, closest points
not: searching, sorting

Ex:

\[ \begin{array}{c}
\text{\(7\)} \rightarrow \text{\(9\)} \rightarrow \text{\(3\)} \rightarrow \text{\(4\)} \rightarrow \text{\(8\)} \\
\text{\(8\)} \rightarrow \text{\(5\)} \rightarrow \text{\(6\)} \rightarrow \text{\(4\)} \rightarrow \text{\(5\)}
\end{array} \]

\[ \text{\(7 + (2) + 5 + (1) + 3 + 4 + 8 = 30\) (Optimal is 27).} \]

Note: Greedy doesn't work!

Brute Force: consider all possible combinations
For each of \(n\) pairs of stations, 2 possibilities \(\Rightarrow O(2^n)\)

Better: consider smaller versions of the problem, use solutions to those to solve larger + larger versions.

Find minimum time through each station first, then go back to find sequence of stations.

\[ \text{Min time through station 1? \(\min(7, 8) = 7\)} \]
Group Work:
- Use these times to find min. time through station 2.
- Find a general formula to find min. time through Station J.

"Consider line 1 for station 2.
Two ways to get there: from Line 1 or from Line 2.

Station 2, Line 1, (2 ways:)
From line 1: $7 + 9 = 16$ \( \rightarrow \) min. time for
" " 2: $8 + 2 + 9 = 19$ \( \rightarrow \) station 2, line 1.

Station 2, Line 2, (2 ways:)
From line 1: $7 + 2 + 5 = 14$ \( \rightarrow \)
" " 2: $8 + 5 = 13$ \( \rightarrow \) min. time for Station 2
line 2

Note: Not always best to come from the same line.

(100) \( \rightarrow \) (1) \( \leftarrow \) best option is to transfer from line 2.
(1) \( \rightarrow \) (1) 101 vs 2

Min time through Station 2: $2 + 9 = 13$. 
"Let's generalize this for any station j."

\[ C(S_{1j}) = \text{fastest time through } S_{1j} \]
\[ C(S_{2j}) = \text{fastest time through } S_{2j} \]

Need to express \( C(S_{1j}) \) and \( C(S_{2j}) \) in terms of previous problem: \( C(S_{1j-1}) \) \( C(S_{2j-1}) \).

(1) Dynamic Programming Formulation

\[ C(S_{1j}) = \]

1. \[ \min \{ \]
   \[ \begin{align*}
   &1: & C(S_{1j-1}) + 0 + a_{1j} &\text{if } j > 1 \\
   &2: & C(S_{2j-1}) + t_{2j-1} + a_{1j} 
   \end{align*} \]

Which one?

For which values of \( j \)? \( j = 2, 3, \ldots, n \)

For \( j = 1 \)? \( C(S_{11}) = a_{11} \) (2) Base Case